EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2000; **29**: 37–62

Compensation of time-delay for control of civil engineering structures

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SUMMARY

Time-delay is an important issue in structural control. Applications of unsynchronized control forces due to time-delay may result in a degradation of the control performance and it may even render the controlled structures to be unstable. In this paper, a state-of-the-art review for available methods of time-delay compensation is presented. Then, five methods for the compensation of fixed time-delay are presented and investigated for active control of civil engineering structures. These include the recursive response method, state-augmented compensation method, controllability based stabilization method, the Smith predictor method and the Pade approximation method, all are applicable to any control algorithm to be used for controlled design. Numerical simulations have been conducted for MDOF building models equipped with an active control system to demonstrate the stability and control performance of these time-delay compensation methods. Finally, the stability and performance of the phase shift method, that is well-known in civil engineering applications, have also been critically evaluated through numerical simulations. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: civil engineering structures; time-delay compensation

INTRODUCTION

Active, semi-active and hybrid control of civil engineering structures have been investigated theoretically and experimentally, and different control systems have been installed in buildings, bridges and other structures [1, 2]. Since the entire control process involves measuring response data, computing control forces, transmitting data and signals to actuators, etc., a time-delay in applying control forces to the structure cannot be avoided. Applications of unsynchronized control forces due to time-delay may result in a degradation of the control performance and may even render the controlled structure to be unstable. Hence, the problem of time-delay is important in structural control and it has been investigated quite extensively in the literature [3–6].

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Received 5 April 1999
Revised 22 July 1999
Accepted 29 July 1999

CCC 0098–8847/2000/010037–26$17.50

The time-delay problem can be addressed in two aspects. The first aspect is to investigate the effect of time-delay on the stability and performance of the controlled system. The second aspect is to study the method of compensation for the time-delay in order to improve the performance of the control system. Recently, a literature survey for the effect of time-delay on control of civil engineering structures and a critical evaluation of available analysis methods for determining the critical time-delay have been made by the authors [3]. A general method of analysis for the determination of the critical time-delay for MDOF system was also presented in Agrawal and Yang [3].

The objectives of this paper are to: (1) present a state-of-the-art review of available methods for time-delay compensation with emphasis on civil engineering applications, and (ii) present and investigate five methods of time-delay compensation for control of civil structures subject to seismic excitations. These include the recursive response method [5], the state-augmented compensation method, controllability based stabilization method, the Smith predictor method [6] and the Pade approximation method [6], all are applicable to any control algorithm to be used. Numerical simulations have been conducted for MDOF building models equipped with an active control system to demonstrate the stability and control performance of these time-delay compensation methods. Finally, the stability and performance of the phase shift method, that is well-known in civil engineering applications, have been evaluated thoroughly through numerical simulations and its limitations are pointed out.

PROBLEM FORMULATION

The equation of motion for an n-degree-of-freedom building structure subject to a horizontal earthquake ground acceleration $\ddot{x}_0$ can be written as

$$\bar{M}\ddot{X}(t) + \bar{C}\dot{X}(t) + \bar{K}X(t) = \bar{H}V(t) + \xi\ddot{x}_0(t) \tag{1}$$

in which $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is an n-vector with $x_i(t)$ being the interstory drift of the ith story unit, $\bar{M}$, $\bar{C}$ and $\bar{K}$ are $(n \times n)$ mass, damping and stiffness matrices, respectively, $V(t) = [v_1, v_2, \ldots, v_r]^{\mathrm{T}}$ is a r-vector of actual control forces applied to the structure, $\bar{H}$ is the controller location matrix, and ξ is the earthquake excitation influence vector. In the state space, Equation (1) can be written as

$$\dot{Z}(t) = AZ(t) + BV(t) + E(t), \quad Z(0) = Z_0 \tag{2}$$

where Z is a $2n$ state vector, A and B are $(2n \times 2n)$ and $(2n \times r)$ matrices, respectively, ad $E(t)$ is a $2n$ vector given by

$$Z(t) = \begin{bmatrix} X \\ \dot{X} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & \mathrm{I} \\ -\bar{M}^{-1}\bar{K} & -\bar{M}^{-1}\bar{C} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \bar{M}^{-1}\bar{H} \end{bmatrix}, \quad E(t) = \begin{bmatrix} 0 \\ \bar{M}^{-1}\xi\ddot{x}_0(t) \end{bmatrix}$$

For the system in Equation (2), the output vector $y(t)$, is defined as

$$y(t) = CZ(t) \tag{3}$$

where C is a $(m \times 2n)$ observation matrix, and m is the number of sensors.

In structural control, an ideal control law $U(t)$ may be designed to calculate control forces based on the measurement of output. However, the actual control force $V(t)$ applied to the structure may be different from the ideal control force because of time-delay. Consider an ideal linear feedback control law $U(t)$ obtained by neglecting the time-delay,

$$U(t) = -Gy(t) \tag{4}$$

In reality, however, the dynamic system involves a time-delay β. Hence, the control force $V(t)$ applied to the structure at the time t is

$$V(t) = U(t-\beta) = -Gy(t-\beta) \tag{5}$$

If the unsynchronized control $V(t) = U(t-\beta) = -Gy(t-\beta)$ is used, the stability of the closed-loop system in Equation (2) is not guaranteed and the performance of the controlled structure may be degraded to an unacceptable level. Various methods of time-delay compensation have been proposed in the literature. A brief literature review for methods of time-delay compensation is presented in the following.

LITERATURE REVIEW

The compensation of time-delay in controlled systems has been a subject of research in other areas of engineering for a few decades [7–14]. For civil engineering applications, Abdel-Rohman [15] investigated a method of time-delay compensation by expanding $V(t) = U(t-\beta)$ in Equation (2) into a Taylor series and augmenting it to Equation (2) [10, 11]. The resulting augmented system does not have any delay term and it can be used to design a controller using conventional methods of controller design. Consequently, the control force applied to the structure at time t is obtained as

$$V(t) = U(t-\beta) = -G_z Z(t-\beta) - G_{u1} U(t-2\beta) - G_{u2} \dot{U}(t-2\beta) \tag{6}$$

in which G_z, G_{u1} and G_{u2} are appropriate gain matrices. It is observed from Equation (6) that, in addition to delayed state vector $Z(t-\beta)$, the controller also requires the past control force $U(t-2\beta)$ and rate $\dot{U}(t-2\beta)$ for the calculation of the control force applied at time t. Abdel-Rohman [15] used the pole assignment method to investigate such a method of time-delay compensation numerically. His results show that a larger amount of control force is required to achieve the same level of response reduction as obtained for the ideal system without time-delay, i.e. $\beta = 0$. Since the Taylor series expansion is based on the assumption that the time-delay β is much smaller than the fundamental period of the structure, the application of this method may be limited to small time-delays only. Moreover, such a method does not guarantee the stability of the controlled structure. Abdel–Rohman [16] further studied the delay compensation for a distributed parameter system using velocity feedback based on the same approach. His results show that the number of vibrational modes that can be compensated for time-delay is equal to the number of actuators, if a low pass filter is used to filter out frequency components of higher modes from the velocity signal.

Chung *et al.* [17–19], McGreevy *et al.* [20] and Soong [21] proposed the phase-shift method of time-delay compensation for a SDOF system. This method is based on the idea of changing the phase angle between the control force and structural response to obtain a good response reduction [22]. In this method, feedback gains $\bar{g}_1$ and $\bar{g}_2$ for the delayed system, i.e.

$u(t) = \bar{g}_1 x(t-\beta) + \bar{g}_2 \dot{x}(t-\beta)$, are obtained from that of the delay-free system, i.e. $u(t) = g_1 x(t) + g_2 \dot{x}(t)$, such that closed-loop systems with and without time-delay have the same active stiffness and active damping. The performance of this approach has been investigated experimentally [20]. Their results clearly show that the method performs well when the time-delay compensation is made using the dominant frequency of excitation. However, the performance degrades when the compensation is made using the structural frequency. Chung *et al.* [19] and Reinhorn *et al.* [23] extended the phase-shift method to MDOF systems by the modal expansion based on the modeshapes and natural frequencies of the open-loop system. The effectiveness of the phase-shift method for particular feedback gains has also been shown through experimental results. However, the stability and performance of the phase-shift method is not guaranteed [3–6].

Pu and Kelly [24] presented a slightly different version of the phase-shift compensation method. In their approach, time-delay compensated gains are obtained such that forward feedback transfer functions are the same for systems with and without time-delay at the poles of the ideal system (without time-delay).

McGreevy *et al.* [20] presented a time-delay compensation approach based on the prediction of $Z(t)$ using past measurements of $X(t-\beta)$, $\dot{X}(t-\beta)$ and $\ddot{X}(t-\beta)$ by a Taylor series expansion. Although this method was successful in compensating time-delay for a SDOF system [20], it was unsuccessful in compensating time-delay for a three-storey building experimentally [19].

Rodellar *et al.* [25, 26] presented predictive algorithms for the time-delay compensation. In their approach, a discrete-time system is derived for the continuous-time system in Equation (2) to predict the response at any instant $k+j$ based on the response at k and the control sequence $U(k+j-1-d)$, $j = 1, 2, \ldots, \lambda + d$, where $d = \beta/\Delta t$ = time-delay expressed as a multiple of the sampling period Δt and λ = prediction horizon parameter (an integer). This predictive system is used to derive a predictive controller by minimizing a LQR performance index; with the result,

$$V(k) = U(k-d) = G_s Z(k-d) + \sum_{i=1}^{d} G_{ui} U(k-d-i) \tag{7}$$

where G_s and G_{ui} ($i = 1, 2, \ldots, d$) are appropriate gain. For a SDOF system equipped with an active tendon system, it was shown experimentally that the predictive control methods works well.

Abdel-Mooty and Roorda [27] studied the control of a simply supported beam experimentally. It was shown that a serious instability problem existed because of time lag in the control system. Hence, a practical scheme was proposed based on the response sampling, curve fitting in a predetermined time function, and then extrapolating to account for the delay in the response measurement. The applicability of this method was demonstrated experimentally for a simply supported beam subject to sinusoidal excitations. However, this method may not be suitable for civil structures subject to stochastic excitations.

Agrawal *et al.* [28, 29] presented a method to compensate time-delay by modelling the time-delay as a transportation lag due to the flow of control signal through a hypothetical pipeline of unit length. Physically, this pipeline can be understood as the flow of signal through different components between sensors to actuators. Dividing the time-delay β into q equal intervals $\Delta\alpha = \beta/q$ along the unit length of the pipeline, q finite-difference equations can be written for the flow of the control force signal $U(t - i\,\Delta\alpha)$ through q segments of the pipeline. These q equations can be combined with the system equation, Equation (2) with $V(t) = U(t-\beta)$, to obtain a delay

free augmented system. Then, the augmented system can be used to design a controller using conventional control methods; with the result

$$V(t) = U(t-\beta) = -G_z Z(t-\beta) - \sum_{i=1}^{q} G_{ui} U(t - i\,\Delta\alpha - \beta) \tag{8}$$

where G_z and G_{ui} ($i = 1, 2, \dots, q$) are appropriate gains. It is observed from Equation (8) that the control force applied to the structure at time t required the state measurement at time $t-\beta$ and control force history during the time interval $t-\beta$ to $t-2\beta$. The performance of the controller in Equation (8) was investigated using the classical LQR approach. It was shown that the control performance improves with the increase of the number of discretizations (q). For a particular example of SDOF system, the required number of discretizations (q) has been found to be approximately 30. The infinite-dimensional problem for the controller in Equation (8), when the number of discretizations approaches infinity, was originally investigated by Soliman *et al.* [30]. In their approach, the solutions of the state and control gain matrices in Equation (8) were obtained in terms of partial differential equations, which are generally difficult to solve.

Sain *et al.* [31] presented the method of Padé approximation for the time-delay compensation of a SDOF systems using LQR controller. In this method, time-delay is represented by the input–output delay of a fictitious system, whereas the transfer function of such a system is $e^{-s\beta}$. Then, the fictitious system is augmented to the structural system, Equation (2), and the controller is designed based on the augmented system.

Inaudi and Kelly [32] proposed a static-output controller to compensate the time-delay as

$$V(t) = U(t-\beta) = -Gy(t-\beta) - \sum_{k=1}^{N_u} \gamma_k U(t-\beta-\tau_k) \tag{9}$$

where G is the output feedback gain matrix, $y(t) = CZ(t)$ is the output vector (e.g. velocity of floors in Equation (3)), γ_k ($k = 1, 2, \dots, N_u$) are control gains, and $\tau_k = k\beta/N_u$. In Equation (9), the parameters γ_k ($k = 1, 2, \dots, N_u$) are calculated such that the $2n$ poles of the feedback path transfer function for the time-delayed controller $U(t-\beta)$ in Equation (9) match those of the closed-loop poles of the system $(A - BGC)$ for the ideal controller $U(t) = -Gy(t)$.

Note that a time-delayed system has infinite number of poles. The calculation of the gains γ_k ($k = 1, 2, \dots, N_u$) by matching poles of the time-delayed and the ideal system guarantees only $2n$ poles of the time-delayed system in the left-half complex plane. Other poles of the time-delayed system most likely will move to the right-half of the complex plane, making the closed-loop system unstable. We have verified this observation through numerical simulations. Our numerical results show that the example in Inaudi and Kelly [32] is unstable, because other poles have moved to the right-half complex plane, and Equation (9) may be physically unrealizable.

Chung *et al.* [33] presented two methods of time-delay compensation using the discrete controller design formulation. The first method of time-delay compensation is similar to the method proposed by Day and Hsia [34], in which LQR controller is first solved for an undelayed system and then the delays are taken into account by using forward substitution of the discretized system equation. Following this approach, Chung *et al.* [33] derived a time-delay compensated LQR controller by minimizing a quadratic performance index; with the result

$$V(k) = U(k-d) = -\bar{G}Z(k-d), \quad \bar{G} = (2R + B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PAS^{d} \tag{10}$$

in which

$$S = \{I - B(2R + B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}P\}A, \quad P = A^{\mathrm{T}}PA + 2Q - A^{\mathrm{T}}PB(2R + B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA \tag{11}$$

For this method, the stability of the controlled structure is not guaranteed.

In another method proposed by Chung *et al.* [33], an augmented discrete system is formed by combining the discrete equation of Equation (2) with the discrete equations for past state vectors $Z(k-1), Z(k-2), \ldots, Z(k-d)$. This augmented system is used to design a time-delay compensated controller based on the static-output LQR approach [35]

$$V(t) = -\bar{G}\bar{Y}(k), \quad \bar{Y}(k) = C_a\bar{Z}(k) = Z(k-d) \tag{12}$$

Static output controller is derived in Equation (12) because only the output $y(k) = Z(k-d)$ is available. One drawback of this approach is the large size of augmented matrices associated with the iterative solutions of nonlinear equations in Reference [35]. For example, for a ten-storey building with 10 ms, time-delay and 1000 Hz sampling frequency, the size of augmented system matrix will be (220×220). As a result, the iterative solutions for the controller in Equation (12) may suffer numerical convergence or stability problems.

Qi and Kuang [36] presented a time-delay compensation method based on the extended linear Kalman filter. Through the numerical simulation results for an eight-storey building equipped with an AMD system, they showed that the performance of this method is not affected by the amount of time delay in the system.

Studies described so far have focused on the deterimental effect of time-delay. Udwadia and Kumar [37], Kumar [38], and Kumar and Udwadia [39] investigated the advantages of appropriate choices of time delays for stabilizing noncollocated control of certain classes of classically damped structural systems. They showed that undamped systems, which cannot be stabilized by use of noncollocated sensors with pure velocity feedback, could be stabilized by using time-delayed velocity feedback control. Further, they obtained easy-to-compute analytical expressions for the bounds on time delays in order to maintain the small gain stability. These results are important for velocity feedback control of large structures.

The stabilization of the system in Equation (2) with $V(t) = U(t-\beta)$ has been investigated by various researchers in other areas of engineering [12, 40, 41]. It has been shown that the system in Equation (2) with $V(t) = U(t-\beta)$ is completely stabilizable by the following dynamic memory controller:

$$U(t-\beta) = -GZ(t-\beta) - \int_{-\beta}^{0} G\mathrm{e}^{-A(s+\beta)}BU(t-\beta+s)\,\mathrm{d}s \tag{13}$$

where G is a $(r \times 2n)$ feedback gain matrix. Using the results presented by Soliman *et al.* [30], Lewis [12] has shown that the gain matrix G in Equation (13) can be obtained by solving the following coupled nonlinear equations:

$$A^{\mathrm{T}}E_0 + E_0A - G^{\mathrm{T}}RG + Q = 0 \tag{14}$$

$$G = R^{-1}B^{\mathrm{T}}E_1, \qquad E_1 = \mathrm{e}^{A^{\mathrm{T}}\beta}E_0 + L\mathrm{e}^{-A\beta} \tag{15}$$

$$LA + A^{\mathrm{T}}L = G^{\mathrm{T}}RG - \mathrm{e}^{A^{\mathrm{T}}\beta}G^{\mathrm{T}}RG\mathrm{e}^{A\beta} \tag{16}$$

where Q and R are weighting matrices for the states and control, respectively. For the special case in which $\beta = 0$, it is observed from Equations (15) and (16) that $L = 0$ and $E_1 = E_0$, and hence Equations (14)–(16) reduce to

$$A^{\mathrm{T}}E_0 + E_0A - E_0BR^{-1}B^{\mathrm{T}}E_0 + Q = 0, \quad G = R^{-1}B^{\mathrm{T}}E_0 \tag{17}$$

Equation (17) is the well-known algebraic Riccati matrix equation for the design of delay-free LQR controller. Hence, the classical LQR is a special case of Equations (14)–(16) for $\beta = 0$. However, these equations are highly nonlinear in nature and will have to be solved iteratively. Further, there is no guarantee that a convergent solution to these equations exists, and during iterations the solution may diverge if either $G^{\mathrm{T}}RG - Q \geqslant 0$ or $G^{\mathrm{T}}RG - \mathrm{e}^{A^{\mathrm{T}}\beta}G^{\mathrm{T}}RG\mathrm{e}^{A\beta} \geqslant 0$. An optimal solution may exist only for some selected cases of the weighting matrices Q and R as well as the time-delay β.

Most of the references discussed above are related to time-delay compensation for civil engineering structures. The history of time-delay compensation problem is old and an enormous numbers of references are available in other areas of engineering. Although the difference-differential equation for civil engineering structures is defined in Equation (2) with $V(t) = U(t - \beta)$, the general equation for a time-delayed processes can be represented by

$$\dot{Z}(t) = \sum_{i=0}^{n_x} A_i Z(t - \beta_{xi}) + \sum_{i=0}^{n_u} B_i U(t - \beta_{ui}) \tag{18}$$

where A_i and B_i are system and controller matrices for the ith component of the system with delays in states and controllers as β_{xi} and β_{ui}, respectively. Systems with delay in states occur commonly in control of chemical processes, where various states may be accompanied by different time delays due to transportation of materials from one component of the process to another. In contrast to this, the systems for civil engineering structures contain delay in the control action only. This delay occurs because of delay in measurements, the effect of actuator dynamics, filtering, etc. Hence, only the references investigating the compensation of delay in control have been reviewed in this study. Detailed information about different aspects of delays in states is available in Malek-Zavarei and Jamshidi [14].

Time-delay problem has been investigated extensively in other engineering systems, such as chemical, electrical, mechanical and aerospace systems and processes. Smith [7, 8] proposed a novel compensation method, in which a delay-free controller is derived by comparing the closed-loop transfer functions of systems with and without time-delays. Hiratsuka *et al.* [42] proposed the use of first-order partial differential equations for processes having transportation lags. Soliman *et al.* [30] derived an optimal controller for linear systems having transportation lags based on the classical control theory using arbitrarily large number of partial differential equations proposed by Hiratsuka *et al.* [42]. Ross [43] presented a compensated controller involving the solution of a set of partial differential equations. Hammerström and Gros [13] have discussed different methods of delay compensation in terms of approximations, generality, implementation and control quality. They have investigated both continuous-time and discrete-time methods for time-delay compensation proposed by other investigators [34, 43] and have recommended the method proposed by Day and Hsia [34] for the design of a suitable control law. Other time-delay compensation methods that may be of interest to control of civil engineering structures are: linear predictor control [41, 44, 45], Robust control of time-delay systems [46–49], sliding mode control [50–52], optimal control [13, 30, 53–57], Feedback stabilization [58].

SOME NEW RESULTS ON COMPENSATION OF TIME DELAY

To date significant progress has been made in the compensation of time-delay for controlled structures. However, the stability and performance of various methods should be investigated and evaluated. In this paper, the stability and performance of some existing methods, e.g. phase-shift method, Pade approximation method, etc. are critically evaluated and some new methods of time-delay compensation are presented. These new results are described in the following.

Stability and performance of the phase-shift compensation method

Consider a SDOF system without any delay in control $u(t)$ (i.e. $\beta = 0$),

$$m_s\ddot{x}(t) + c_s\dot{x}(t) + k_s x(t) = u(t) - \ddot{x}_0(t) \tag{19}$$

where m_s, c_s and k_s are mass, damping and stiffness coefficients, respectively. An ideal linear controller (i.e. $\beta = 0$) is designed as

$$u(t) = -g_1 x(t) - g_2 \dot{x}(t) \tag{20}$$

where g_1 and g_2 are feedback gains. If time-delays in displacement and velocity measurements are denoted by β_x and $\beta_{\dot{x}}$, respectively, then the actual control force $u(t)$ without compensation, that is applied to the structure, is given by

$$u(t) = -g_1 x(t - \beta_x) - g_2 \dot{x}(t - \beta_{\dot{x}}) \tag{21}$$

The basis of the phase-shift method is to modify feedback gains g_1 and g_2 for the delay-free controller, Equation (21), into $\bar{g}_1$ and $\bar{g}_2$, i.e. $u(t) = -\bar{g}_1 x(t-\beta_x) - \bar{g}_2\dot{x}(t-\beta_{\dot{x}})$, such that both closed-loop systems (with and without delay) have the same active stiffness and active damping. This results in a relationship between delay-free gains and time-delayed gains, for the harmonic motion at the frequency ω, as [18, 19]

$$\begin{bmatrix} \bar{g}_1 \\ \bar{g}_2 \end{bmatrix} = \begin{bmatrix} \cos(\omega\beta_x) & \omega\sin(\omega\beta_{\dot{x}}) \\ \sin(\omega\beta_x)/\omega & \cos(\omega\beta_{\dot{x}}) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{22}$$

Chung *et al.* [19] and Reinhorn *et al.* [23] have extended the phase-shift method to MDOF systems by transforming Equations (1), (3), (4) and (5) with $C = \mathrm{I}_{2n} =$ a $(2n \times 2n)$ identity matrix into the modal domain using the mode-shapes of the open-loop system. The uncontrolled structure in Equation (1) is assumed to be classically damped so that the normal modes exist. Let ω_i and ζ_i be the ith natural frequency and damping ratio of the uncontrolled structure in the ith mode, respectively, and Γ be the $(n \times n)$ real modal matrix consisting of n modeshapes. Further, let $X = \Gamma\eta$ where $\eta = [\eta_1, \eta_2, \ldots, \eta_n]'$ is the modal vector. Substituting $X = \Gamma\eta$ into Equations (3) and (4) with $C = I_{2n}$, the control force applied to the ith mode can be written as

$$\bar{u}_i(t) = -g_{1i}\eta_i(t) - \mathrm{g}_{2i}\dot{\eta}_i(t) \tag{23}$$

where g_{1i} and g_{2i} are feedback gains for modal displacement and velocity, respectively, in the ith mode. In deriving Equation (23), all the coupling terms among different modes due to the feedback control have been neglected, i.e. all the off-diagonal terms of matrices $\Gamma^{\mathrm{T}}\bar{M}^{-1}\bar{H}G_1\Gamma$ and

$\Gamma^{\mathrm{T}}\bar{M}^{-1}\bar{H}G_2\Gamma$ have been neglected. Based on the derivation presented for a SDOF system, the modified modal gains $\bar{g}_{1i}$ and $\bar{g}_{2i}$ for the time-delayed controller for $\omega = \omega_i$ and $\beta_x = \beta_{\dot{x}} = \beta$, i.e.

$$\bar{u}_i(t) = -\bar{g}_{1i}\eta_i(t-\beta) - \bar{g}_{2i}\dot{\eta}_i(t-\beta) \tag{24}$$

can be obtained from Equation (22) after $\bar{g}_{1i}$ and $\bar{g}_{2i}$ are obtained for all the modes $i = 1, 2, \dots, n$, the modified gain matrix in the modal domain is transformed back to the physical domain using the modeshapes of the open-loop system to derive the full-state feedback controller in physical domain as

$$V(t) = -\bar{G}Z(t-\beta) \tag{25}$$

where $\bar{G}$ is the modified feedback gain matrix for the estimated time-delay β, which may or may not be identical to the actual time-delay β_c. A step-by-step algorithm for application of the phase-shift method to MDOF systems is presented in Reinhorn *et al.* [23].

As observed from the above derivations, the modified gain matrix $\bar{G}$ is calculated based on the modeshapes and natural frequencies of the open-loop system. These quantities can be significantly different from those of the closed-loop system with the controller in Equation (25), particularly for large values of time-delays or high values of active damping ratios. As a result, stability of the controlled structure is not guaranteed. The phase-shift method is applicable to any control theory, however, it is not applicable to static output feedback. The stability and performance of the phase-shift method was investigated by Agrawal and Yang [3–6]. The numerical results indicate that the phase-shift compensated controller becomes unstable when the time-delay is large or the active damping is high (i.e. high gain for velocity feedback). The performance of such a method will be critically evaluated in the numerical simulations to be presented later.

Recursive response compensation method

The 'recursive response' method was proposed by Agrawal and Yang [5] in which a controller $U(t) = -GZ(t)$ is designed for the ideal system in Equation (2), i.e. $\beta = 0$. Then, a time-delay compensated gain matrix $\bar{G}$, i.e. $U(t-\beta) = -\bar{G}Z(t-\beta)$ in Equation (25), is derived from G. For the ideal system, the response vector $Z(t)$ can be obtained by integrating the closed-loop system without external excitation $E(t)$ as

$$Z(t) = \mathrm{e}^{(A-BG)t}Z_0 \tag{26}$$

From Equation (26), a recursive relationship between the response $Z(t)$ at time t and the time-delayed response $Z(t-\beta)$ at time $t-\beta$ can be written as

$$Z(t) = \mathrm{e}^{(A-BG)\beta}Z(t-\beta) \tag{27}$$

Hence, $U(t) = -GZ(t) = -G\mathrm{e}^{(A-BG)\beta}Z(t-\beta)$, and the time-delay compensated gain matrix, $\bar{G}$, is obtained as

$$\bar{G} = G\mathrm{e}^{(A-BG)\beta} \tag{28}$$

The modified gain matrix $\bar{G}$ in Equation (28) is derived using the recursive response equation in Equation (27) and it does not guarantee the stability of the closed-loop system. In this paper, the

stability and performance of the recursive response method will be investigated through a numerical example of a three-storey building. Numerical results clearly demonstrate that the stability and performance of the recursive-response method is superior to that of the phase-shift method.

State-augmented compensation method

We propose to discretize the time delay β into q equal intervals of $\Delta = \beta/q$. Then, q backward or central finite-difference equations for the states $Z(t)$ can be written as

$$\dot{Z}(t - j\Delta) = \frac{1}{\Delta}[Z(t-(j-1)\Delta) - Z(t - j\Delta)], \quad j = 1, 2, \ldots, q \tag{29a}$$

or

$$\dot{Z}(t - j\Delta) = \frac{1}{2\Delta}[Z(t-(j-1)\Delta) - Z(t-(j+1)\Delta)], \quad j = 1, 2, \ldots, q \tag{29b}$$

The q equations in Equation (29) are combined with the system equation in Equation (2) to obtain the augmented system as

$$\dot{\hat{Z}}(t) = \hat{A}\hat{Z}(t) + \hat{B}V(t) \tag{30}$$

in which $\hat{Z}(t)$ is the augmented vector of dimension $n_a = 2n(1+q)$,

$$\hat{Z}(t) = [Z^T(t), Z^T(t-\Delta), \ldots, Z^T(t-j\Delta), \ldots, Z^T(t-\beta)]^T \tag{31}$$

and $\hat{A}$ and $\hat{B}$ are $(n_a \times n_a)$ and $(n_a \times r)$ augmented system matrix and controller location matrix, respectively. When central difference equations, Equation (29b), are used to form the augmented system in Equation (30), the backward difference equation, Equation (29a), should be used for the qth equation to satisfy the boundary condition. For the augmented system in Equation (30), only the measurement output $Z(t-\beta)$ is available, and hence $y(t) = Z(t-\beta) = C_a\hat{Z}(t)$ where $C_a = [0_{2n+2n\times(q-1)}, I_{2n}]$. Then, a static output time delay compensated controller $V(t)$ can be derived as

$$V(t) = -\bar{G}y(t) = -\bar{G}Z(t-\beta) \tag{32}$$

The gain matrix $\bar{G}$ for the time-delay compensated controller in Equation (26) can be calculated using any control method, such as static-output LQR or pole-placement approaches, etc. (e.g. Levine and Athans [35], Davison [59]). In this paper, stability and performance of the controller in Equation (32) will be investigated using the static-output pole-placement approach. The results of numerical simulations indicate that a stable time-delay compensated controller can be designed using smaller number of discretizations, q.

Controllability based stabilization method

The gain matrix G of the controller in Equation (13) can be derived using the absolute controllability theorem for time-delayed systems. Stabilization of the system in Equation (2) and $V(t) = U(t-\beta)$ through absolute controllability concept has been investigated by Olbrot [60],

Lewis [12], Kwon and Pearson [40], and Furukawa and Shimemura [41]. Absolute controllability of a system implies that any initial state Z_0 can be transferred to any state $Z(t_1) = 0$ by applying a control, which is null in the interval $[t_1 - \beta, t_1]$. The state vector $Z(t)$ remains zero if no further control is applied. Hence, absolute controllability not only implies the controllability but also stabilizability of the system in Equation (2) with $V(t) = U(t - \beta)$. Olbrot [60] has shown that the system in Equation (2) is absolutely controllable if and only if

$$\mathrm{rank}[D, AD, A^2D, \ldots, A^{n-1}D] = 2n, \quad D = \mathrm{e}^{-A\beta}B \tag{33}$$

It should be noted that the absolute controllability condition above is the same as the controllability condition for an ordinary system having a system matrix A and a controller location matrix D, i.e.

$$\dot{\bar{Z}}(t) = A\bar{Z}(t) + DU(t) + E(t) \tag{34}$$

Hence, a controller designed using the ordinary system in Equation (34) will stabilize the system in Equation (2) with $V(t) = U(t - \beta)$. Lewis [12] has shown that the states $\bar{Z}(t)$ of the ordinary system, Equation (34), related to the system in Equation (2) with $V(t) = U(t - \beta)$ through the relationship

$$\bar{Z}(t) = Z(t) + \int_{-\beta}^{0} \mathrm{e}^{-As} DU(t + s)\, \mathrm{d}s \tag{35}$$

Now, suppose the ordinary system in Equation (34) is used to design a linear controller as

$$U(t) = -G\bar{Z}(t) \tag{36}$$

Then, substitution of Equation (35) into Equation (36) leads to the controller in Equation (13). Dimensions of matrices A and D in the ordinary system in Equation (34) are the same as those of matrices A and B, respectively, in Equation (2). Thus, the controller gain matrix G is derived without solving the nonlinear equations, Equations (14)–(16). This method of time-delay compensation is suboptimal in nature.

Smith's predictor compensation method

Smith [7, 8] proposed a novel method of time-delay compensation, known as Smith's Predictor method, by comparing the transfer functions of closed-loop systems with and without time delay. For the system in Equation (2) with $V(t) = U(t - \beta)$, the transfer functions for $y(s)/U(s)$ follows from Equations (2)–(4) as

$$y(s)/U(s) = H_p(s) = CH(s)B\mathrm{e}^{-s\beta}, \quad H(s) = (sI - A)^{-1} \tag{37}$$

For the ideal system, i.e. $\beta = 0$, the block diagram for the closed-loop system is shown in Figure 1 where $r(s)$ and $x_a(s)$ denote the Laplace transformation of the reference input and the ground acceleration, respectively, and $H_L(s) = H(s)E$. The closed-loop transfer function from the block diagram in Figure 1 for the control system without the disturbance $x_a(s)$ can be obtained as

$$M(s) = y(s)/r(s) = [I + CH(s)G]^{-1}CH(s)G \tag{38}$$

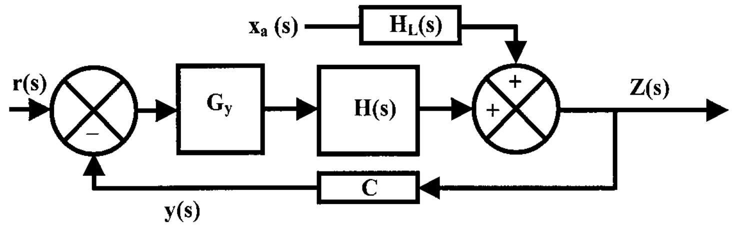


Figure 1. Block diagram of the ideal (without time-delay) closed-loop.

Now, suppose a compensator $\bar{G}(s)$ is designed for the time-delay system, i.e. $U(s) = \bar{G}(s)y(s)e^{-s\beta}$. The closed-loop transfer function for the time-delay system with the compensator $\bar{G}(s)$ can be obtained as

$$M^*(s) = [I + CH(s)\bar{G}e^{-s\beta}]^{-1}CH(s)\bar{G}e^{-s\beta} \tag{39}$$

In the Smith's method, the compensator $\bar{G}(s)$ is designed such that $M^*(s) = M(s)e^{-s\beta}$, indicating that the delayed output $y(t-\beta)$ of $M^*(s)$ is the same output $y(t)$ of the ideal system $M(s)$ (without excitation $x_a(s)$), except a time shift of β. Equating $M^*(s)$ to $M(s)e^{-s\beta}$, one obtains

$$\bar{G}(s) = [I + GCH(s)\{I - e^{-s\beta}\}]^{-1}G \tag{40}$$

The physical representation of the compensator in Equation (40) is shown in Figure 2. It is observed from Equation (40) and Figure 2 that the implementation of the delay compensator $\bar{G}(s)$ requires the delay free compensator, G, the structural model $H(s)$, and the time-delay β. As a result, the performance of the Smith's predictor, $\bar{G}(s)$, may be adversely affected in the presence of uncertainties in $H(s)$ and β.

For civil engineering structures, which are inherently stable, the disturbance rejection capability of the Smith's predictor can be investigated through the transfer function between the output $y(s)$ and the earthquake ground acceleration $x_a(s)$ in Figures 1 and 2. For the ideal system in Figure 1, the transfer function $y(s)/x_a(s)$ is obtained as

$$y(s)/x_a(s) = [I + CH(s)G]^{-1}CH_L(s) \tag{41}$$

Similarly, the transfer function $y(s)/x_a(s)$ for the time-delayed system with the Smith's predictor in Figure 2 is obtained as

$$y(s)/x_a(s) = [I + CH(s)G]^{-1}[I + CH(s)G - CH(s)Ge^{-s\beta}]CH_L(s) \tag{42}$$

When $\beta = 0$, the transfer function in Equation (42) is the same as that in Equation (41) for the ideal system. Further, the magnitude of Equation (42) is quite close to that of Equation (41) when the time-delay β is very small. However, as the magnitude of the time-delay β is increased, the magnitude of Equation (42) will differ significantly from that of Equation (41) and the performance of Smith's predictor will degrade.

It is observed from Equations (41) and (42) that the characteristic equation of the delayed system with Smith's predictor is the same as that of the ideal system, i.e. $|I + CH(s)G| = 0$. Hence,

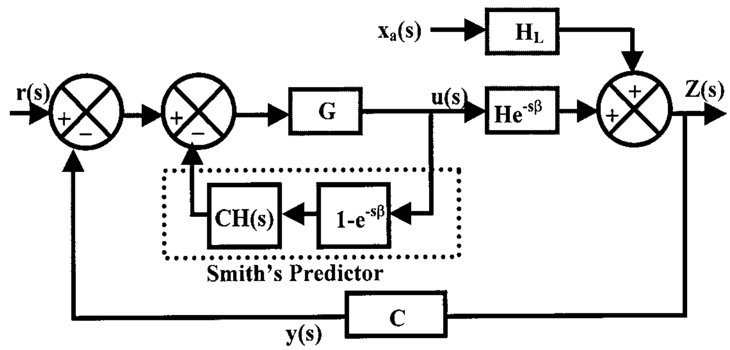


Figure 2. Block diagram of the structure with delayed controller and Smith's predictor.

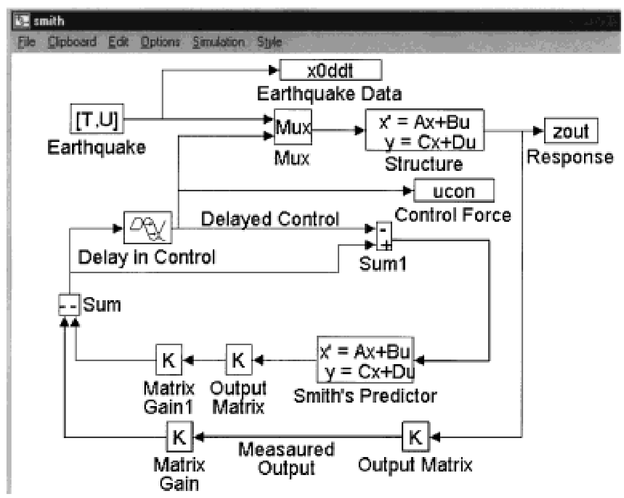


Figure 3. Simulink block diagram of the structure with Smith's predictor.

the stability of the structure using the Smith's predictor is always guaranteed. It is further observed from Figure 2 that the Smith's predictor can be implemented in the time-domain as the dynamic output compensator [61]

$$U(t) = -Gy(t) - CP(t), \qquad \dot{P}(t) = AP(t) + \mathrm{B}\{\mathrm{U}(t) - U(t-\beta)\} \tag{43}$$

The implementation of the Smith's predictor can be made through MATLAB SIMULINK block diagram in Figure 3.

In this paper, the performance of the Smith's predictor method will be investigated for a 3-storey building subject to earthquake ground motions. Numerical results clearly demonstrate the characteristics of Smith's predictor discussed above [6].

Pade approximation method

Sain *et al.* [31] used the Padé approximation method for the compensation of time-delay for a SDOF system using a LQG controller. In this paper, we shall use the LQR and static-output pole placement controllers to investigate the effectiveness of the method for a MDOF system.

In Padé approximation, the time-delay is represented by the input–output delay of a fictitious system, where the transfer function of such a system is $e^{-s\beta}$. Then, the fictitious system is augmented to the structural system, and the controller is designed based on the augmented system. The transfer function $e^{-s\beta}$ is expanded in terms of a rational algebraic function of the form

$$H_\beta(s) = M(s)/N(s) = [m_0 + m_1 s + \cdots + m_k s^k]/[n_0 + n_1 s + \cdots + n_k s^k] \tag{44}$$

where $M(s)$ and $N(s)$ are kth order polynomials in ascending order of s, and the coefficients m_i and n_i $(i = 0, 1, \ldots, k)$ are given by

$$n_i = [k!(2k-1)!]/[i!\,2k!\,(p-i)!], \qquad m_i = (-1)^i n_i \tag{45}$$

Since the coefficients n_i $(i = 0, 1, \ldots, k)$ in Equation (45) are derived such that the zeros of the polynomial $N(s)$ are in the left complex plane, the Padé approximation is stable. The transfer function realization in Equation (45) can be represented by a state-space fictitious system with k states as

$$\dot{z}_p(t) = a_p z_p(t) + b_p U(t), \qquad y_p = U(t-\beta) = c_p z_p(t) + d_p U(t) \tag{46}$$

where a_p, b_p, c_p and d_p are appropriate matrices, which can be derived using the function *pade* in Matlab. It is observed from Equation (46) that the output y_p of the fictitious system with input $U(t)$ is the time-delayed control force $U(t-\beta)$. As a result, Equation (46) can be combined with Equation (2) with $V(t) = U(t-\beta)$ to form a delay-free augmented system as

$$\dot{\bar{Z}}(t) = \bar{A}\bar{Z}(t) + \bar{B}U(t) + \bar{E}(t)\ddot{x}_0(t) \tag{47}$$

where $\bar{A}$, $\bar{B}$ and $\bar{E}$ are appropriate augmented matrices and $\bar{Z} = [Z^{\mathrm{T}}, z_p^{\mathrm{T}}]^{\mathrm{T}}$ is the $(2n+k)$ augmented state vector. The augmented system in Equation (47) does not contain the delay term $U(t-\beta)$ and it can be used to design controllers using conventional control theories. In this paper, we use the LQR and static-output pole placement methods to investigate the performance of such a method for a MDOF system. Numerical results clearly demonstrate that the Padé approximation method is quite effective in compensating the time-delay [6].

NUMERICAL EXAMPLES

The stability and performance of different methods for time-delay compensation described above will be investigated using the 3DOF experimental building model considered by Chung *et al.* [19]. The open-loop frequencies of this building model are 2.24, 6.83 and 11.53 Hz, and the corresponding modal damping ratios are 1.62, 0.39 and 0.36 per cent, respectively. An active tendon control system is installed in the first-storey unit. For the design of a linear (LQR) controller in Equation (2), we choose a (6×6) state weighting matrix Q as: $Q(i,j) = K(i,j)$ for $i = 1, 2, 3$ and $j = 1, 2, 3$ where K is the (3×3) stiffness matrix of the building, and $Q(i,j) = 0$ otherwise. Two types of controllers, denoted by 'Low Gain' and 'High Gain', are designed by using control weighting scalars $R = 42\,480$ and 4248, respectively. The damping ratios for the

three modes of the building without time-delay using the low gain controller are 12.82, 12.33 and 5.48 per cent, respectively. Likewise, the damping ratios using the high gain controller are 26.94, 30 and 16.34 per cent, respectively, which are relatively high.

In the previous formulation, the gain matrix designed based on the ideal system ($\beta = 0$) is denoted by G, referred to as the uncompensated (or unsynchronized) controller, whereas the compensated one is denoted by $\bar{G}$. For simplicity of presentation, a time delay β is used throughout the formulation. In reality, however, the actual time delay β is a statistical variable (i.e. the value is not known precisely), whereas the compensated time delay will be denoted by β_c, that is the best estimate of β. Consequently, all the notations β appeared in the time-delay compensators should be replaced by β_c, e.g. $\bar{G} = \bar{G}(\beta_c)$. In the following numerical examples, such a distinction is important (although not mentioned in the literature), because the stability and performance of some methods of time-delay compensation are quite sensitive to the error in estimating β, i.e. $\beta_c - \beta$.

Stability and performance of phase-shift compensation method

Based on the phase-shift algorithm, the modified gain matrix $\bar{G}(\beta_c)$ in Equation (25) has been computed for $\beta_c = \beta_{ix} = \beta_{ix}$ ($i = 1, 2, 3$) for both low gain and high gain controllers, where the actual time-delay β may not be equal to the compensated time-delay β_c. Suppose $\beta = 0$ and the compensated time-delay $\beta_c = 35$ ms. In this case, it is found that the closed-loop system $(A - B\bar{G})$ is unstable. As a result, the accuracy of the compensated time-delay β_c, that is used for computing the phase-compensated gain matrix, $\bar{G}(\beta_c)$, has a significant bearing on the stability and performance of the controller. Likewise, even if the compensated time-delay β_c is identical to the actual time-delay β, the stability and performance of the phase-compensated controller, Equation (25), should be investigated. In this connection, the stability analysis method presented in Agrawal and Yang [3] is useful.

In Agrawal and Yang [3], the controller is assumed to be stable for the system without a time-delay. Then, the critical time-delay is computed as the minimum time-delay beyond which the system becomes unstable. Using this approach, the phase-compensated controller in Equation (25) can be shown to be stable only in a region of time-delay ($\bar{\beta}_{\min} \leqslant \beta \leqslant \bar{\beta}_{\max}$), where $\bar{\beta}_{\min}$ and $\bar{\beta}_{\max}$ are referred to as the minimum and maximum critical time-delays. It suffices to investigate the stability region ($\bar{\beta}_{\min} \leqslant \beta \leqslant \bar{\beta}_{\max}$) for different compensated time-delay β_c in order to examine the stability of the phase-compensated controller.

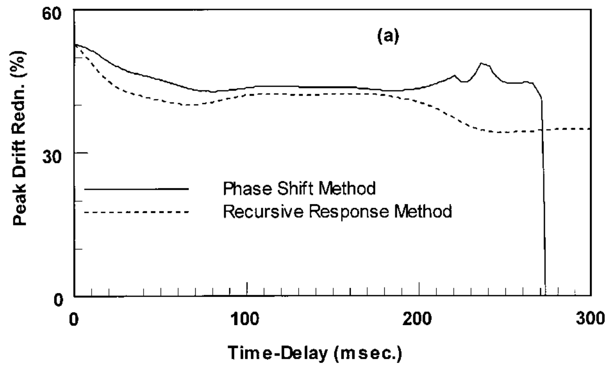
Time-delay compensated gain matrices, $\bar{G}(\beta_c)$, are calculated using the phase-shift method for β_c in the range of 35–250 ms for the low gain controller designed above. Then, $\bar{\beta}_{\min}$ and $\bar{\beta}_{\max}$ are calculated for each gain matrix $\bar{G}(\beta_c)$ using the method of stability analysis described in Agrawal and Yang [3]. The results for the stability region ($\bar{\beta}_{\min}, \bar{\beta}_{\max}$) for different values of β_c are shown in Columns (1) and (2) of Table I. As observed from Table I, for the compensated time-delay β_c in the range of 35 ms to 250 ms, β_c in Column (1) is always within the stability region ($\bar{\beta}_{\min}, \bar{\beta}_{\max}$) in Column (2). This indicates that the closed-loop system using the phase-shift controller is stable, if the actual time-delay is equal to the compensated time-delay, i.e., $\beta = \beta_c$ for $\beta_c \leqslant 250$ ms. However, if the actual time-delay β is outside the stability region ($\bar{\beta}_{\min}, \bar{\beta}_{\max}$), the controller is unstable. It is further observed from Table I that the stability region becomes smaller as β_c is increased, indicating a lesser degree of tolerance for the estimation error in the time-delay. For the case $\beta_c = 250$ ms, $\bar{\beta}_{\max}$ is equal to 251.5 ms, and hence a small estimation error for the time-delay will render the controller to be unstable. It is shown in Table I that the closed-loop system using

Table I. Stable region of time-delay for three-storey building

Comp. delay β_c (ms)	Stable time-delay region ($\bar{\beta}_{min}$, $\bar{\beta}_{max}$) in msec.	
	Phase-shift method	Recursive resp. method
	Low-gain controller	
(1)	(2)	(3)
0	(0.0, 17.7)	(0.00, 17.72)
100	(78.86, 112.79)	(81.75, 121.06)
200	(179.61, 208.40)	(177.70, 216.50)
270	Unstable (267.39–269.22)	(250.30, 290.80)
300	Unstable	(280.36, 321.07)
500	Unstable	(475.69, 526.69)
	High-gain controller	
(4)	(5)	(6)
0	(0.00, 12.20)	(0.00, 12.20)
18	(0.00, 18.37)	(0.00, 35.80)
100	Unstable	(87.65, 116.39)
200	Unstable	(173.85, 212.91)
224	Unstable	Unstable (188.03–220.17)
293	Unstable	(292.74, 315.88)
300	Unstable	(290.22, 326.47)

the phase-compensated controller becomes unstable for the time-delay $\beta \geqslant 270$ ms, even if β_c is identical to the actual time-delay β. It is further observed from Table I for the case $\beta_c = 0$ that the critical time delay for a controller without compensation ($\beta_c = 0$) is 17.7 ms. In other words, the building becomes unstable for an uncompensated controller when the actual time-delay β exceeds 17.7 ms. As observed from Table I, the compensated controller is capable of pushing the critical time-delay to 270 ms, indicating the importance of compensating time-delay in the design of controllers.

The performance of the phase-shift method using the low gain controller is examined by computing the response of the building model subject to the El Centro NS (1940) earthquake with a peak ground acceleration of 98 gal. For simplicity of presentation, we assume that the actual time delay β is equal to the compensated time delay β_c, i.e. $\beta = \beta_c$ in the following. Based on the phase-shift compensated controller in Equation (25), the percentages of reduction for the peak interstory drifts have been calculated for different values of compensated time-delays $\beta_c = \beta$ for the low gain controller. The percentages of the peak drift reduction for the third-storey unit are presented in Figure 4(a). It is observed that, although the peak reductions deteriorate only slightly with the increase of time-delay β, a significant degradation in the performance occurs around $\beta = 200$ ms. As β is further increased, the building becomes unstable around $\beta = 270$ ms. Figure 4(b) shows the plot of the peak control force vs. β. It is observed that the peak control force required by the delay compensated controller in Equation (25) is generally higher than that required by the ideal controller (i.e. $\beta = \beta_c = 0$). Moreover, the peak control force required by the controller in Equation (25) increases significantly for $\beta > 240$ ms.



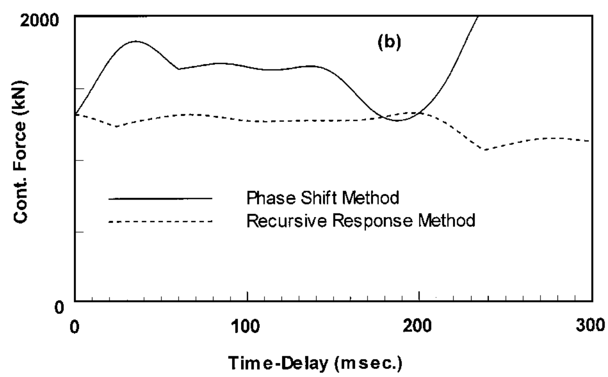


Figure 4. Peak response quantities of three-storey building for different time-delays, $\beta = \beta_c$; (a) reduction of peak interstorey drift of the third storey unit, (b) peak control force.

For the high gain controller, numerical results are shown in Column (5) of Table I. These results indicate that the phase-shift compensated controller is unstable even for a small time-delay $\beta \leqslant 20$ ms. Hence, the phase-shift method fails to compensate small amount of time-delay for high gain controllers. Although the performance of the phase-shift method has been investigated experimentally by Chung *et al.* [19] for small values of time-delay ($\beta = 35$ ms), the numerical results presented in this study demonstrate that the time-delay compensation by the phase-shift method is not reliable and may result in unstable closed-loop system when either the time-delay is large or the level of active damping in various modes is high (i.e. high gain controller).

Stability and performance of the recursive response method

Based on the recursive response method for time-delay compensation in Equation (28), the regions of stable time-delays $(\bar{\beta}_{\min}, \bar{\beta}_{\max})$ are calculated for the delay compensated gain matrix $\bar{G}(\beta_c)$ for the low and high gain controllers described previously. The results are shown in Columns (3) and (6) of Table I for comparison. For the low gain controller, it is observed from Column (3) of Table I that the recursive response method is stable for $\beta = \beta_c > 500$ ms. However,

the phase-shift method becomes unstable for $\beta > 270$ ms. For the high gain controller, it is noticed from Column (5) that the phase-shift method is capable of compensating time delay up to $\beta < 18$ ms. only. On the other hand, the recursive response method is able to compensate the time-delay up to $\beta < 224$ ms, see Column (6). Moreover, the delay-compensated controller by the recursive response method is also stable for $\beta \geqslant 293$ ms. Hence, the recursive response method is capable of stabilizing high gain controllers with a large time-delay.

The response reduction characteristics of the recursive response methods are investigated using the same El-Centro (NS) earthquake as the previous case. Figures 4(a) and (b) show the percentage of peak drift reduction for the third-storey unit and the peak control force, respectively, as a function of time-delay, $\beta = \beta_c$, for the low gain controller. It is observed from Figures 4(a) and (b) that, while the response reduction using the recursive response method is similar to that using the phase-shift method for $\beta < 200$ ms for the low gain controller, the peak control force required by the recursive response method is much smaller than that by the phase-shift method. Consequently, the recursive response method proposed herein is superior to the phase-shift method in terms of the stability and control performance.

Stability and performance of the state-augmented method

The stability and performance of the state-augmented controller in Equation (32) will be demonstrated using the static-output pole placement method [59]. Since the total number of measurements is $2n$, $2n$ poles of the closed-loop augmented system, Equation (30), are placed at prescribed locations. These $2n$ prescribed poles are obtained from the closed-loop system based on the LQR controller obtained previously for low gain and high gain controllers. An important parameter in the design of the state-augmented controller, Equation (32), is the number of discretizations q in Equation (29). For the prescribed locations of poles using the low gain controller, $\bar{G}(\beta_c)$ matrix in Equation (32) was designed for different values of q and a stability analysis was performed using the method presented in Agrawal and Yang [3] to find the stable time-delay region $(\bar{\beta}_{\min}, \bar{\beta}_{\max})$. Figure 5 shows the plots of critical time-delays (i.e. $\beta_{\min}$ and $\beta_{\max}$) vs. the number of discretizations, q, for $\beta_c = 35$, 50 and 100 ms based on the backward difference formulation for augmented states, Equation (29a). It is observed from Figure 5 that the stability region, i.e. $\beta_{\min} < \beta < \beta_{\max}$, widens and becomes asymptotic with an increase of q. Further, β_c lies well within the stability region with $q > 5$ for $\beta_c = 35$ and 50 ms and with $q > 25$ for $\beta_c = 100$ ms. Hence, stable time-delay compensated controllers can be designed by choosing appropriate number of discretizations, q, for a particular value of time-delay. The central difference formulation for the augmented states has also been proposed in Equation (29b). In general, it is superior to the backward difference when the time-delay is large. For instance, for the case of $\beta_c = 100$ ms, the asymptotic stable time-delay region is clearly established for $q = 8$ as shown by the dashed curve in Figure 5. On the other hand, the backward difference formulation requires $q = 18$.

The performance of the state-augmented method in Equation (32) is investigated for the same building and the same earthquake using prescribed locations of poles for low gain and high gain controllers described previously. The gain matrix $\bar{G}$ in Equation (32) is designed for different values of time-delays β for $q = 50$ where $\beta_c = \beta$. Numerical simulations have been conducted using the central difference approach. Figures 6(a) and (b) show plots for the percentages of the peak drift reduction and the peak control force vs. time-delay, $\beta_c = \beta$, respectively, similar to Figure 5. It is observed that the behavior of peak drift reduction is similar to the recursive response method in Figure 5(a). Further, there is only small variation in the peak control force for

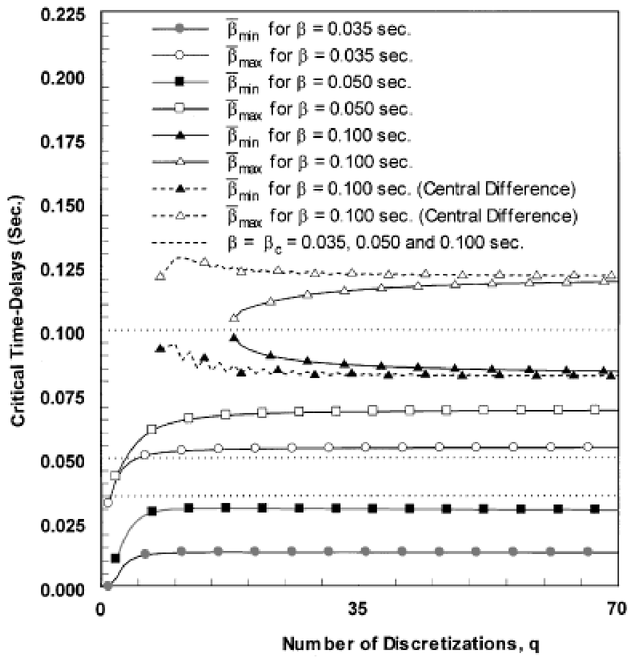


Figure 5. Critical time-delays ($\overline{\beta}_{min}$ and $\overline{\beta}_{max}$) vs. number of discretizations, q, for different values of compensated time-delay β_c for the state-augmented compensation method.

different time-delays β as shown in Figure 6(b). Consequently, the state-augmented approach proposed herein is an effective time-delay compensation method.

Performance of controllability based stabilization method

The time-delay compensated controller based on the controllability based stabilization method as presented in Equations (34)–(36) is theoretically guaranteed to be stable. Hence, it is not necessary to investigate the stability of such a controller. However, its control performance will be investigated using the same example considered previously, i.e. the same 3-storey building model subject to the same earthquake. Again, the compensated time-delay β_c is assumed to be identical to the actual time-delay β. The percentages of reduction for peak interstory drifts (x_1, x_2, x_3), peak control force U, and total control $\overline{U^2}$ for the phase-shift controller for various time-delays are shown in Columns (2)–(6) of Table II, in which the total control energy $\overline{U^2}$ is the integral of the square of the control force over the duration of the earthquake, i.e. 20 s. The closed-loop system with the phase-shift controller becomes unstable for $\beta = 280$ ms.

For the controllability based stabilization controller (CBSC) in Equations (34)–(36), the numerical simulations are conducted based on the LQR method by choosing weighting matrices $\bar{Q}$ and $\bar{R}$ for ordinary system in Equation (34) such that the response reductions and peak control force are almost similar in values as those for the phase-shift method. The percentages of reduction for peak interstory drifts, peak control force and total control energy for CBSC are

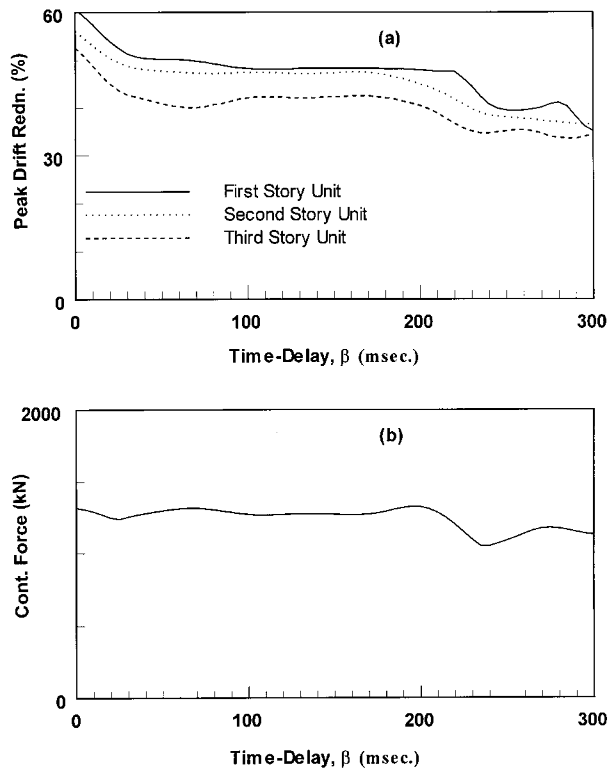


Figure 6. Peak response quantities of three-storey building using state-augmented compensation method; (a) peak drift reduction vs. time-delay, $\beta = \beta_c$; (b) peak control force vs. time-delay, $\beta = \beta_c$.

shown in Columns (7)–(11) of Table II. It is observed from the Table II that although the performance of both methods are similar for small time-delays, the controllability based stabilization method outperforms the phase-shift method for larger time-delays. Further, the stability of CBSC is always guaranteed.

Performance of Smith predictor compensation method

The Smith's Predictor Controller (SPC) has been proven theoretically to be stable and hence we shall not investigate its stability. The gain matrix $\bar{G}(s)$ of SPC in the transformed domain is presented in Equation (40), whereas the SPC is implemented in the time-domain using Equation (43). In this method, a constant gain matrix G for the ideal system $\beta = 0$ is designed first (similar to the phase-shift method, recursive response method, etc.) and the SPC is implemented through Equation (43). To investigate the performance of SPC, the same example considered previously is used herein. For the design of G, we consider the same high gain and low gain LQR controllers used in the example of the phase-shift method. The percentages of peak drift reduction vs. time-delay, β with $\beta_c = \beta$, for the two LQR controllers are presented in Figure 7. It is observed from Figure 7 that the Smith's predictor is capable of maintaining the performance of the ideal

Table II. Percentages of reductions for interstorey drifts (x_1, x_2 and x_3), peak control force (U) and control energy ($\overline{U^2}$) using the controllability based stabilization method and phase-shift method.

Comp. delay, $\beta_c = \beta$	Phase-shift method					Control. based stabilization				
	x_1 (%)	x_2 (%)	x_3 (%)	U (kN)	$\overline{U^2}$ (kN^2)	x_1 (%)	x_2 (%)	x_3 (%)	U (kN)	$\overline{U^2}$ (kN^2)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	61	60	59	1317.7	85.46	61	60	59	1317.7	85.46
35	57	59	60	1817.5	159.4	57	59	57	1817.3	122.7
200	50	54	53	1318.6	98.18	48	50	47	1318.3	80.11
250	35	44	45	2219.6	295.8	46	45	41	1317.3	77.28
280			Unstable			41	42	39	1316.8	85.79

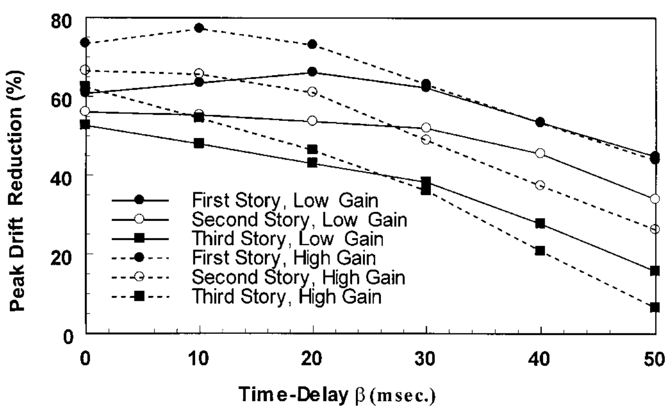


Figure 7. Peak drift reduction vs. time-delay, β, for Smith's predictor method.

system ($\beta = 0$) for $\beta \leqslant 20$ ms. For higher values of time-delay, although the stability of the controlled structure is guaranteed, the performance degrades drastically. This has been expected as shown in Equations (41) and (42). However, the peak control forces for all values of time-delays remain the same as the ideal system. Hence, the Smith's Predictor method is suitable for applications when time-delays are small. An interesting and advantageous property is that both the state and output feedback controllers can be implemented using the Smith's Predictor method.

Performance of Padé approximation method

The time-delay has been modelled by the input–output relation of a fictitious system that is taken into account in the design of the Padé Approximation Controller (PAC). Hence, the PAC is theoretically stable, and we shall investigate its performance using the same numerical example considered previously.

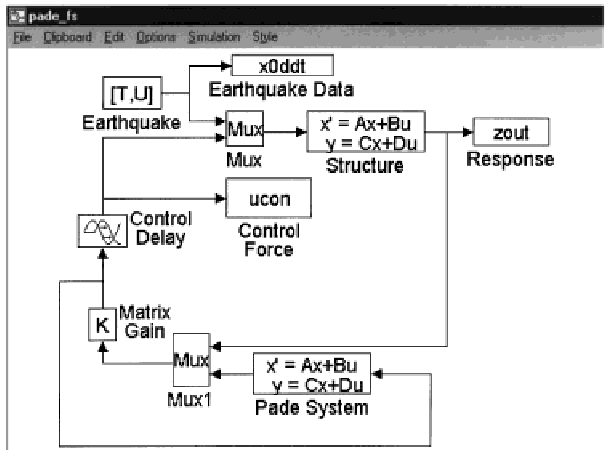


Figure 8. Simulink block diagram of structure with complete state feedback controller using Padé approximation method.

The order of Padé approximation, k, is an important parameter. Simulations were conducted for different values of time-delays and the Padé approximation order k. For time-delays $\beta(=\beta_c)$ less than 100 ms, a third order Padé approximation was found to be sufficient. For larger time-delays, a higher-order Padé approximation was required, depending on the stability of the transfer function in Equation (44). The augmented state vector $\bar{Z}$, Equation (47), consists of the states Z of the structure and fictitious states z_p of the Padé approximation. Hence, two types of controllers, denoted by 'Complete State Feedback (CSF)' and 'Full State Feedback (FSF)' are considered. A CSF controller is designed by using the LQR method as

$$u(t) = -G\bar{Z}(t), \qquad G = -R^{-1}\bar{B}^{\mathrm{T}}P, \qquad P\bar{A} + \bar{A}^{\mathrm{T}}P + \bar{Q} - P\bar{B}R^{-1}\bar{B}^{\mathrm{T}}P = 0 \tag{48}$$

in which the measurements of Z and $z_p(t)$ are required. $z_p(t)$ is obtained by the online integration of the Padé approximation, Equation (46), as shown in Figure 8. Two cases of CSF controllers are designed by choosing the state weighting matrix $\bar{Q}$ such that the first (3×3) elements of $\bar{Q}$ are the same as K, i.e. the stiffness matrix of the structure, and other elements of $\bar{Q}$ are zero. Further, $R = 42\,480$ and 4248 are considered. Figure 9 shows the percentages of peak drift reductions vs. the time-delay β with $\beta_c = \beta$, for the two CSF controllers. It is observed that the PAC is able to compensate time-delay with small degradation for the peak drifts. Further, it has been noted that the level of the peak control force is approximately the same for all values of time-delay, i.e. $1885 < u_{\max} < 1963$ for $R = 42\,480$, and $1871 < u_{\max} < 2124$ for $R = 4248$. It is further observed from Figure 9 that the performance for peak drift reductions for the high gain and low gain controllers are quite similar.

A full-state feedback (FSF) controller requires the measurements of only Z but not $z_p(t)$. Hence a static-output controller is designed as

$$u(t) = -\bar{G}C_y\bar{Z}(t) = -\bar{G}Z(t), \quad C_y = \lfloor I_{(2n\times 2n)} \;\; 0_{(k\times k)} \rfloor \tag{49}$$

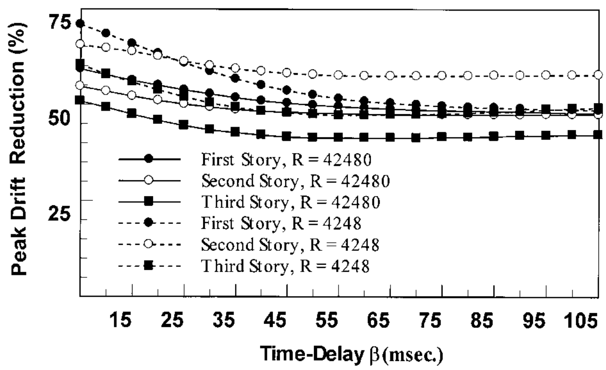


Figure 9. Peak drift reduction vs. time-delay, β, for CSF controllers based on Padé approximation method.

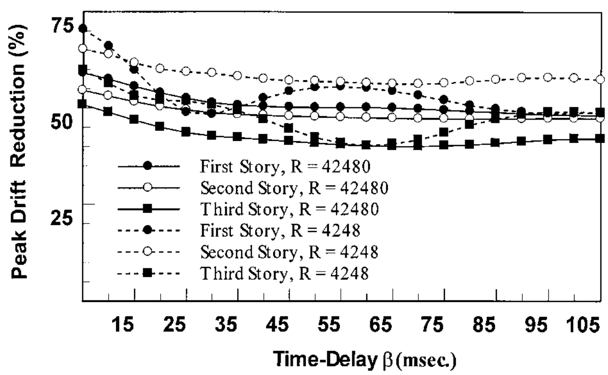


Figure 10. Peak drift reduction vs. time-delay, β, for FSF controllers based on Padé approximation method.

The FSF controller is a full-state feedback controller with respect to the original system, Equation (1), and an output feedback controller with respect to the augmented system, Equation (47). Hence, any output feedback algorithm [35] can be used to find the controller gain matrix $\bar{G}$. The iterative procedure of Levine and Athans [35] failed to converge in the design of the FSF controller for Equation (47). Hence, the static output pole placement algorithm of Davison [59] was used to find gain matrix $\bar{G}$ for Equation (47). Since the total measurements in this case are 6 (velocities and displacements), 6 prescribed locations of poles were obtained from the closed-loop system $(A - BG)$, where G is the gain matrix for the two (high gain and low gain) LQR controllers previously designed for the ideal system ($\beta = 0$) for the phase-shift method. Figure 10 shows the percentages of peak drift reduction vs. the time-delay $\beta(=\beta_c)$ for the two FSF controllers. It is observed from Figure 10 that the performance of two FSF controllers are quite similar to those of the CSF controllers, Figure 9. Further, the peak control forces for CSF and FSF controllers are also quite similar.

CONCLUSIONS

Available methods of time-delay compensation have been reviewed and evaluated. Five methods for the compensation of fixed time-delay are presented and investigated for active control of civil engineering structures. These include the recursive response method, the state-augmented compensation method, the controllability based stabilization method, the Smith predictor method and the Padé approximation method, all are applicable to any control algorithm to be used for controller design. The stability and performance of the phase shift method, that is well-known in civil engineering applications, have also been critically evaluated.

Numerical results demonstrate that the time-delay compensation by the phase-shift method is not reliable and may result in unstable closed-loop system when either the time-delay is large or the level of active damping in various modes is high (i.e. high gain controller). On the other hand, the recursive response method and the state-augmented compensated method proposed herein can be used to modify the gain matrix, similar to the phase-shift method, but these methods are superior to the phase-shift method in terms of stability and control performance. Likewise, the controllability based stabilization method outperforms the phase-shift method for larger time-delays, and its stability is always guaranteed. The Smith's Predictor method is guaranteed to be stable and it is capable of compensating small time-delays without much performance degradation. However, a significant performance degradation occurs when time-delays are large. The Padé approximation method is guaranteed to be stable and it is shown to be capable of compensating for all values of time-delay with only a small performance degradation.

Finally, most of the time-delay compensation methods reviewed and presented in this paper are applicable to either state feedback or static output feedback controllers. Among the five methods presented herein, only the Padé approximation method is applicable to dynamic output feedback controllers. Dynamic output controllers have important practical applications, for instance, the entire control system of civil engineering structures is installed with only a limited number of acceleration sensors. In this regard, further research is needed to develop stable and effective time-delay compensation methods for dynamic output feedback controllers.

ACKNOWLEDGEMENT

This research is partially supported by National Science Foundation Grant Nos. CMS 96-25616 and CMS 96-15731.

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